Mathematics of Per-Pixel Lighting

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Overview

- **Why Per-Pixel Lighting?**
- **Review**
  - **OpenGL Transforms and Spaces**
  - **OpenGL Per-vertex Lighting**
  - *Object Space* **Per-vertex Lighting**
- ***Surface-local Space*?**
  - **Other names**
  - **Why is this necessary?**
  - *Surface-local Space* **Per-Vertex Lighting**
- **Per-Pixel Lighting**
  - **In** *Surface-local Space*
  - **In other spaces?**

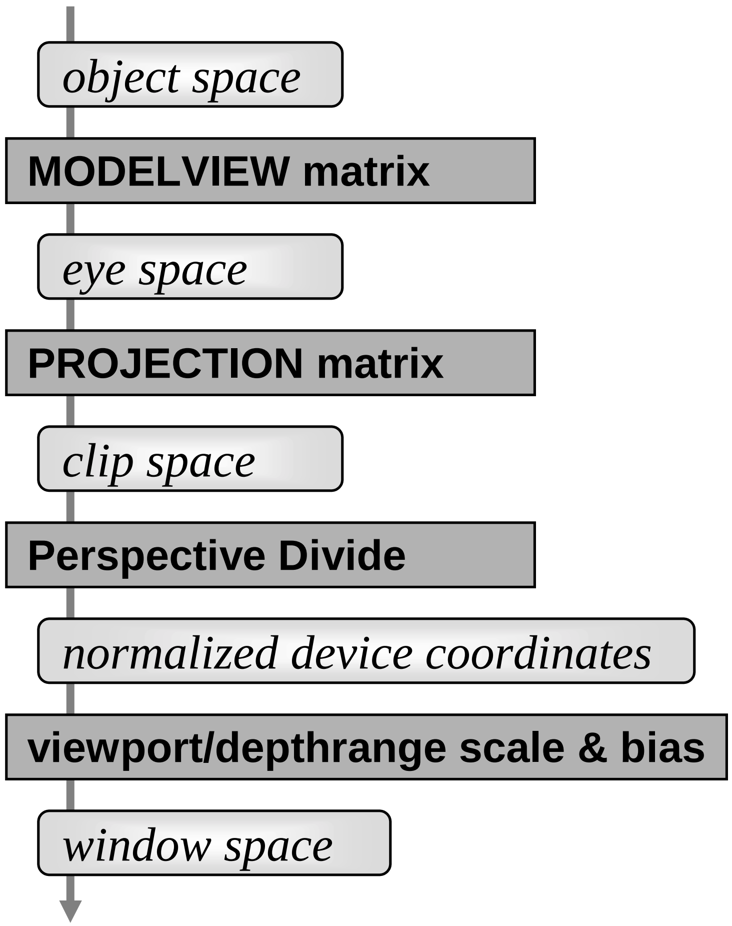
Why Per-Pixel Lighting?

- Because it looks better than per-vertex lighting
- Because it's hardware accelerated
- Because everyone else is doing it
  - Don't be the last on your block

This is do-it-yourself lighting

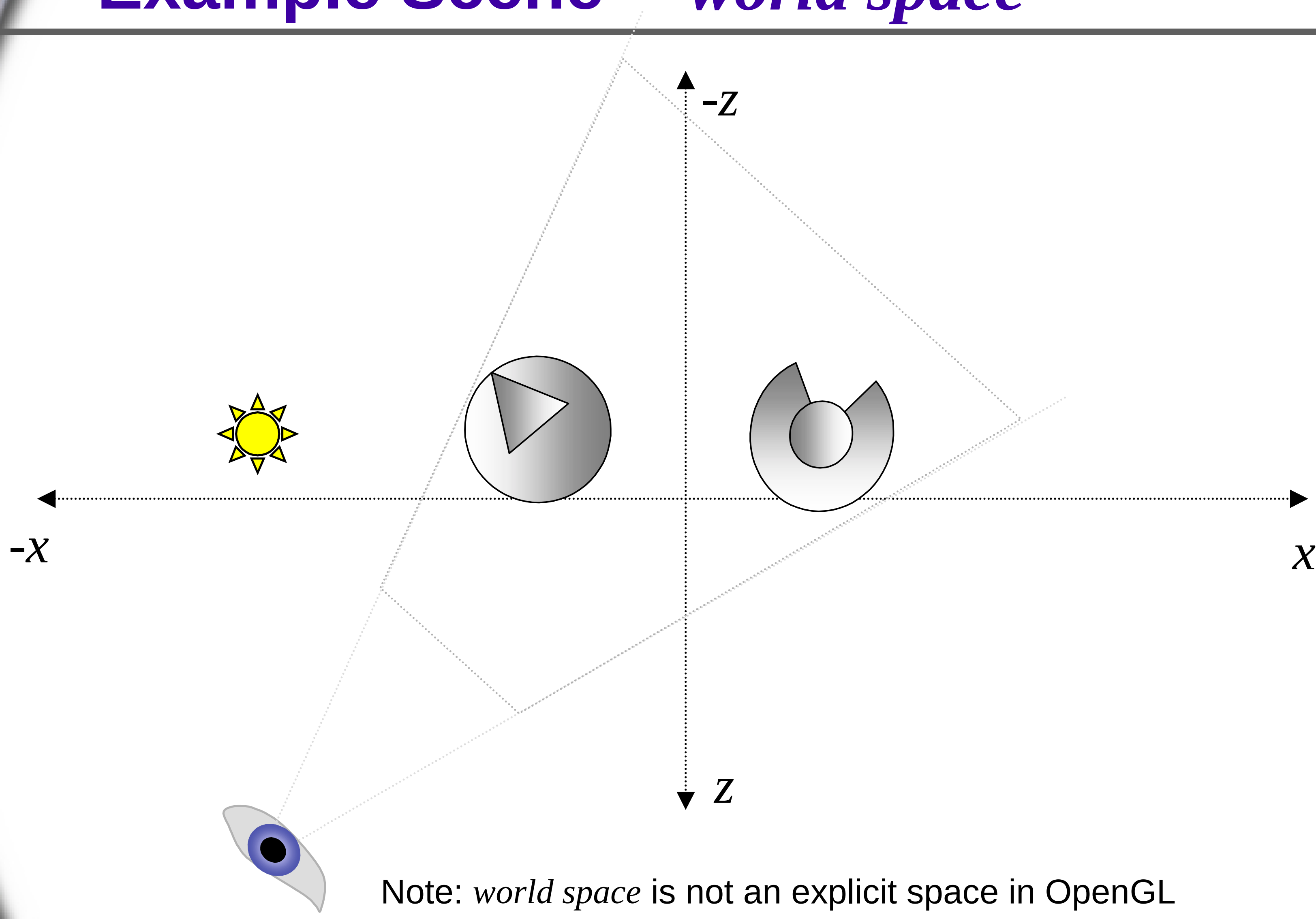
- You get total control, but this means you have to do it all
  - No `glShadeModel(GL_PHONG)`
  - No `glEnable(GL_BUMP_MAPPING)`
- If you don't know how to implement per-vertex lighting, learn how to do that first
- Per-pixel shading is an extension of per-vertex shading (for the most part)

OpenGL Transformations



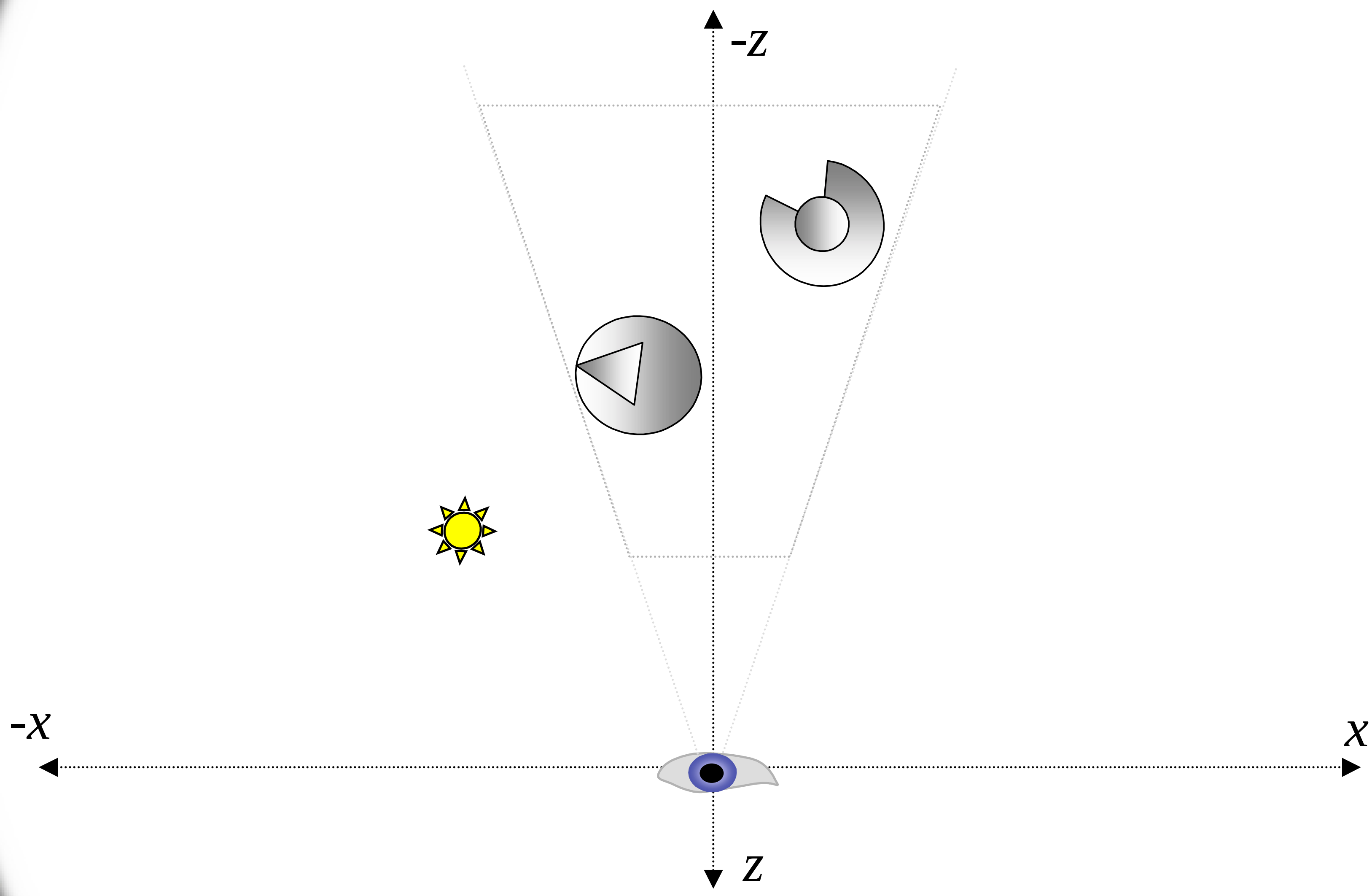
- **OpenGL operation transforms coordinates through several** *coordinate frames* **or** *spaces*
- **Each of the** *spaces* **has various properties that make it useful for some operation**
- **Vertex attributes are specified in** *object space*
- **Lighting, eye-linear texgen, and fog happen in** *eye space*
- **Clipping happens after projection in** *clip space*
- **Rasterization happens in** *window space*

Example Scene -- *world space*



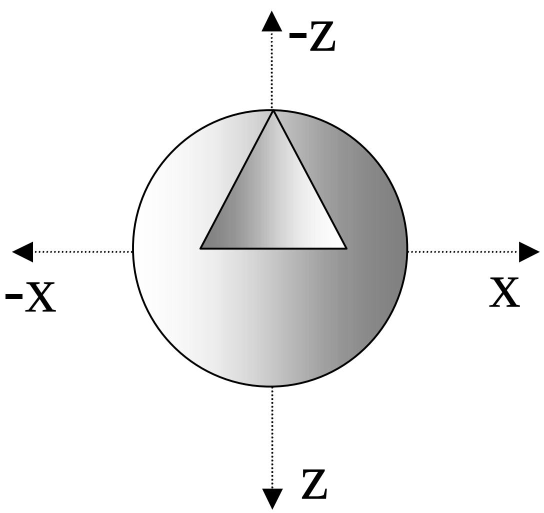
Note: *world space* is not an explicit space in OpenGL

Example Scene -- *eye space*

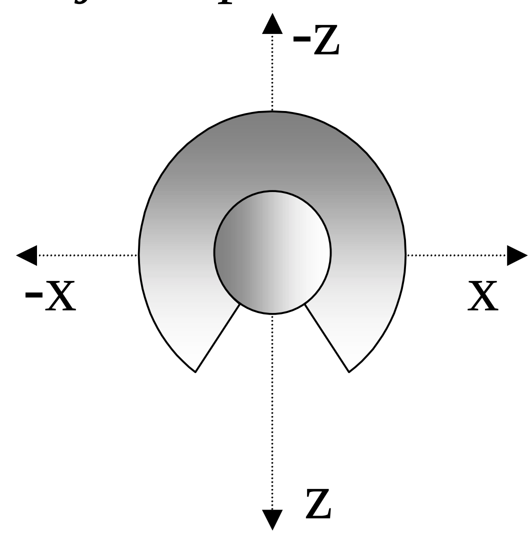


Object Space

Object Space For



Object Space For



Each object has its own origin, orientation, and scale

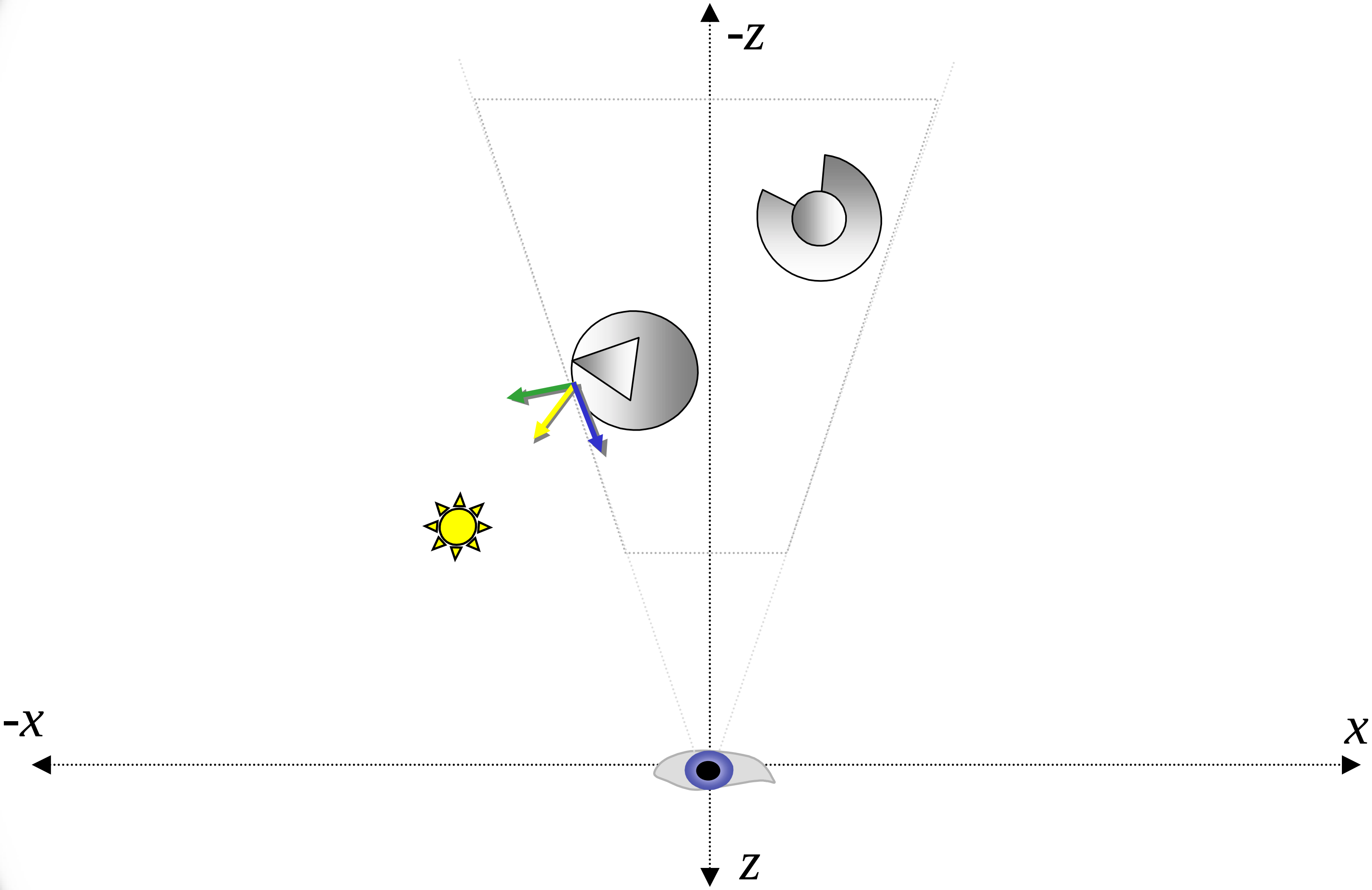
OpenGL Per-Vertex Lighting

- **For OpenGL Per-Vertex Lighting, all calculations happen in** *eye space*
- **Not essential, but convenient**
- **For each OpenGL per-vertex light, the illumination is computed as (assuming separate specular)**

$$C_{pri} = (spot)(att)\left[a_{cm}a_{cli} + (\mathbf{n} \bullet \mathbf{l})d_{cm}d_{cli}\right]$$

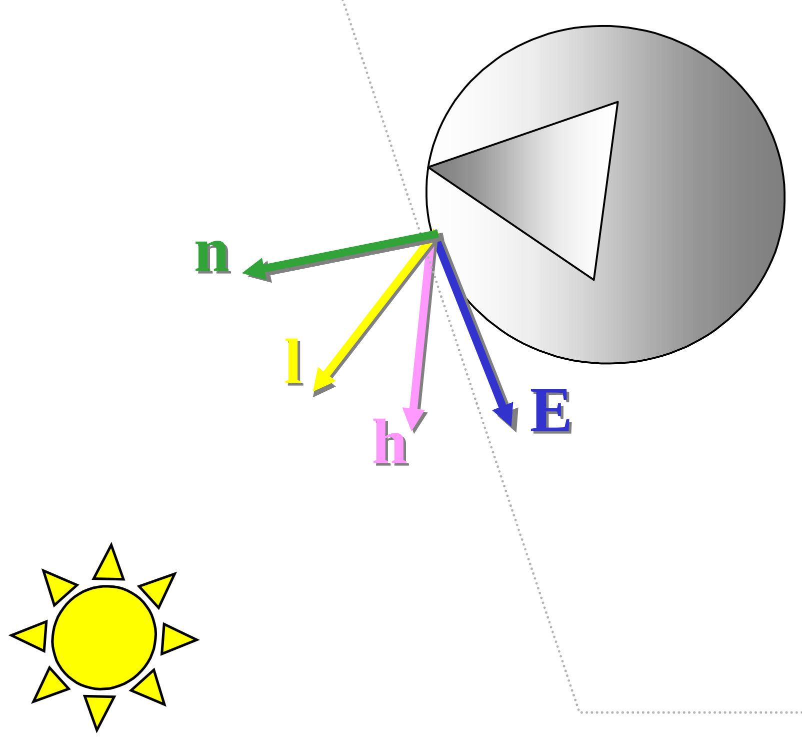
$$C_{sec} = (spot)(att)(f)(\mathbf{n} \bullet \mathbf{h})^{s_{rm}} s_{cm}s_{cli}$$

Lighting in *eye space*



Lighting in *eye space* (2)

The vectors…

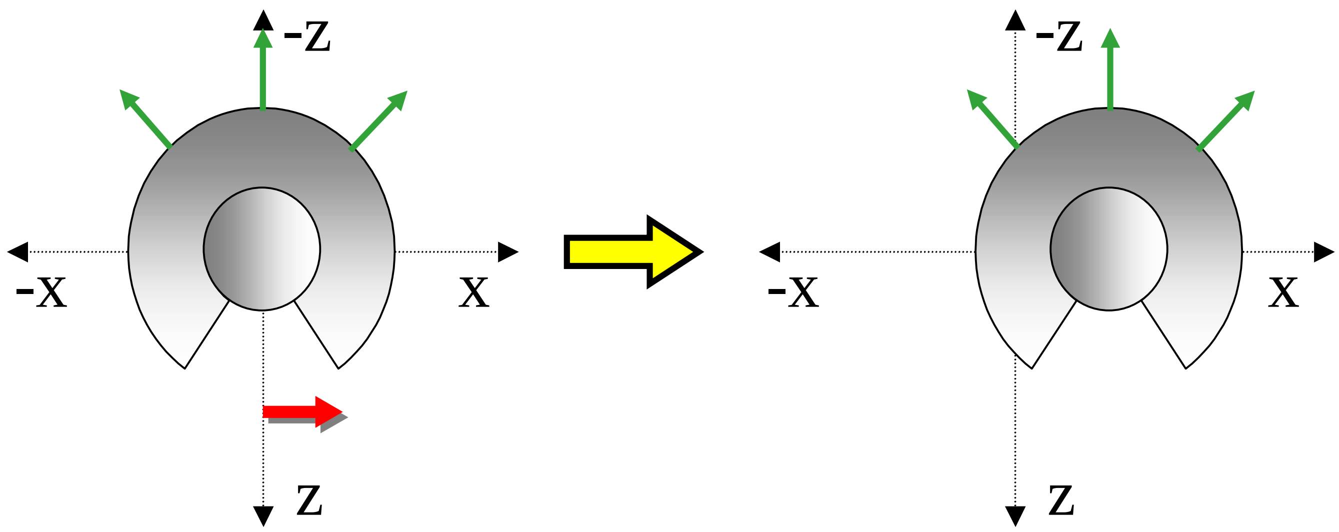


Transforming Normals

- **To evaluate the lighting equation in** *eye space***, normals must be transformed from** *object space* **into** *eye space*
- **Normals are not simply transformed by the modelview matrix like position**
- **You may know from the Red Book or various other sources that "normals are transformed by the inverse-transpose of the modelview matrix", but let's consider why…**
- **The following slides should help provide some intuition about the transforming of normals**

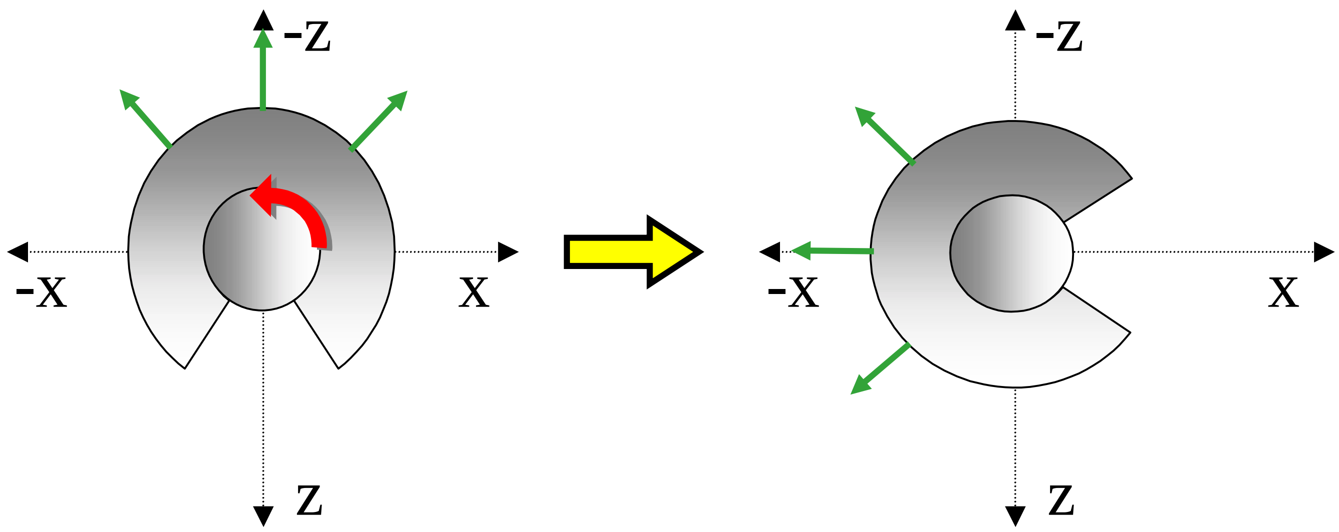
Transforming Normals (2)

- **Translation of position does not affect normals**



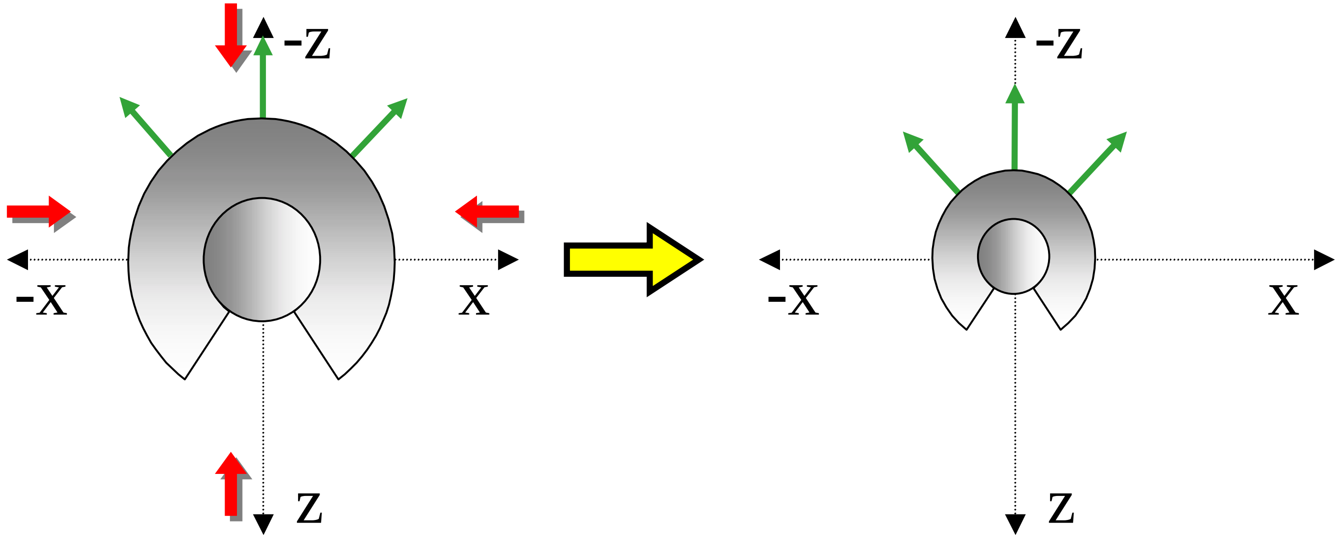
Transforming Normals (3)

- Rotation is applied to normals just like it is to position



Transforming Normals (4)

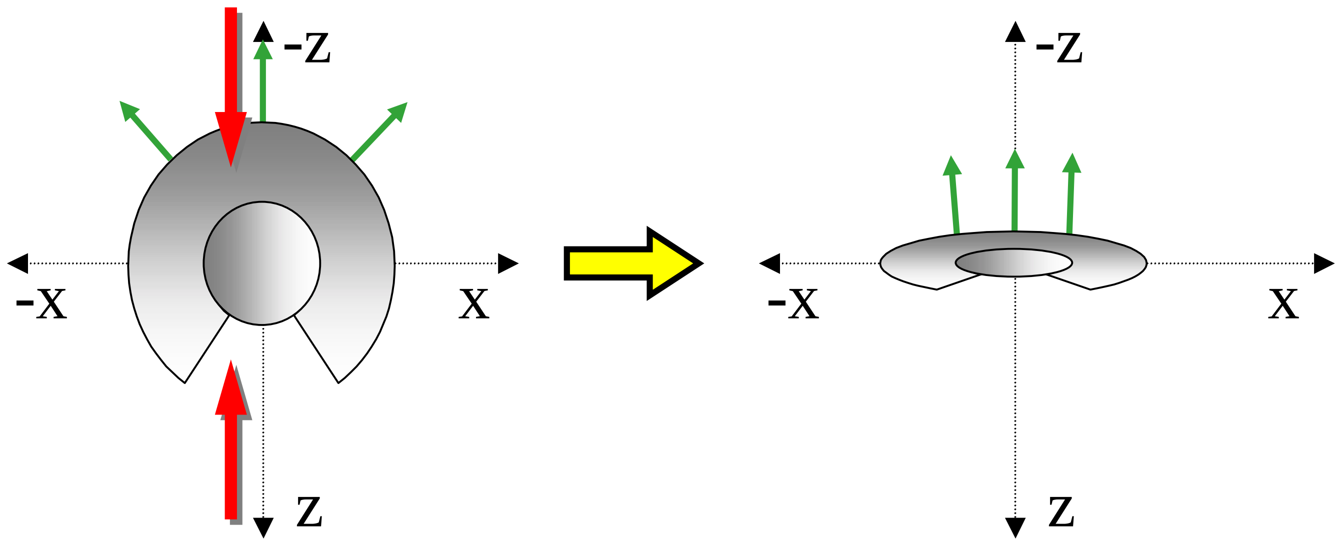
- **Uniform scaling of position does not affect the direction of normals**



Note that we are *only* considering how the *direction* of a normal is affected by transforming the position

Transforming Normals (5)

- **Non-uniform scaling of position does affect the direction of normals!**
  - ***<u>Opposite</u>* of the way position is affected – or the *<u>inverse</u>* of the scaling matrix that's applied to position**



Note that we are *only* considering how the *direction* of a normal is affected by transforming the position

Transforming Normals (6)

- **To summarize, these are the basic position transformations and the corresponding normal transformation:**

	position	normal
translation	**T**	**I**
rotation	**R**	**R**
scaling	**S**	$\mathbf{S}^{-1}$

- **Note that any sort of scaling applies inversely to the normal – we treat all scales (uniform and non-uniform) the same**
- **This is why we need GL_NORMALIZE and GL_RESCALE_NORMAL for OpenGL lighting**
- **We have to deal with it in per-pixel lighting as well**

Transforming Normals (7)

- **How does this match what OpenGL does?**

$$\boldsymbol{n}_e = \mathbf{M}^{-\mathrm{T}} \boldsymbol{n}_o$$

- **For simplicity, consider $\mathbf{M}$, the modelview matrix, is composed of a scale and a rotation**
  - *inverse-transpose* **is distributive**
  - **For rotation (orthonormal) matrices** $\mathbf{R}^{-1} = \mathbf{R}^{\mathrm{T}}$**, and** $\mathbf{R}^{-\mathrm{T}} = \mathbf{R}$
  - **For scaling (diagonal) matrices** $\mathbf{S} = \mathbf{S}^{\mathrm{T}}$

$$\begin{aligned} \mathbf{M}^{-\mathrm{T}} &= (\mathbf{RS})^{-\mathrm{T}} \\ &= \mathbf{R}^{-\mathrm{T}} \mathbf{S}^{-\mathrm{T}} \\ &= \mathbf{RS}^{-1} \end{aligned}$$

This matches our ad hoc result!

Object Space Per-Vertex Lighting

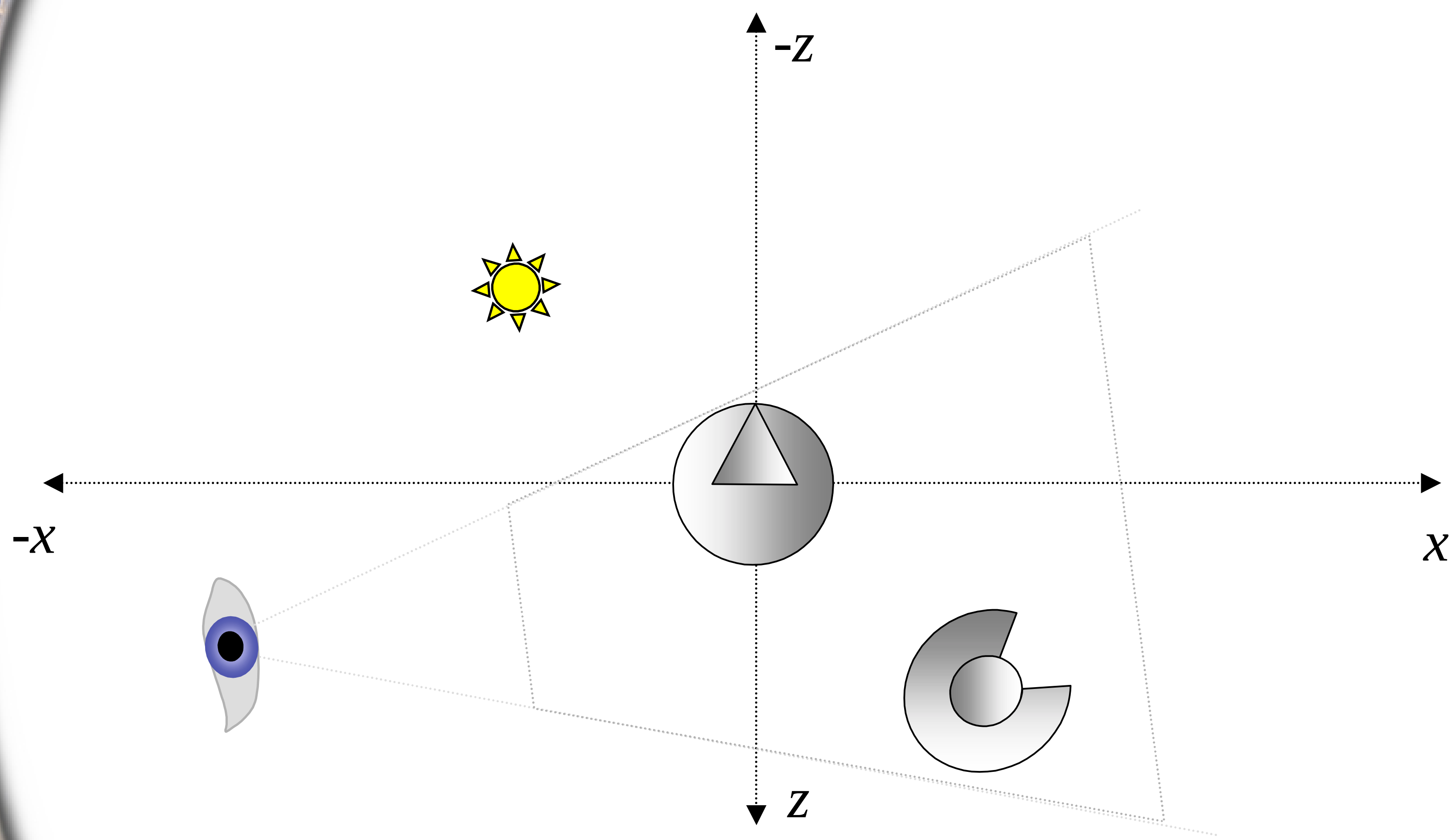
- **Nothing in the lighting equation requires evaluation in** *eye space* **- consider lighting in** *object space* **instead**
  - **Non-uniform scaling in the modeling matrix would complicate things, so we will ignore that for now…**
- **If the modeling matrix is simply a rigid body transform, then this is easy…**
  - **Need to transform the light into** *object space* **from eye space**

$$\boldsymbol{l}_{obj} = \mathbf{M}^{-1}\boldsymbol{l}_{eye}$$ local light source

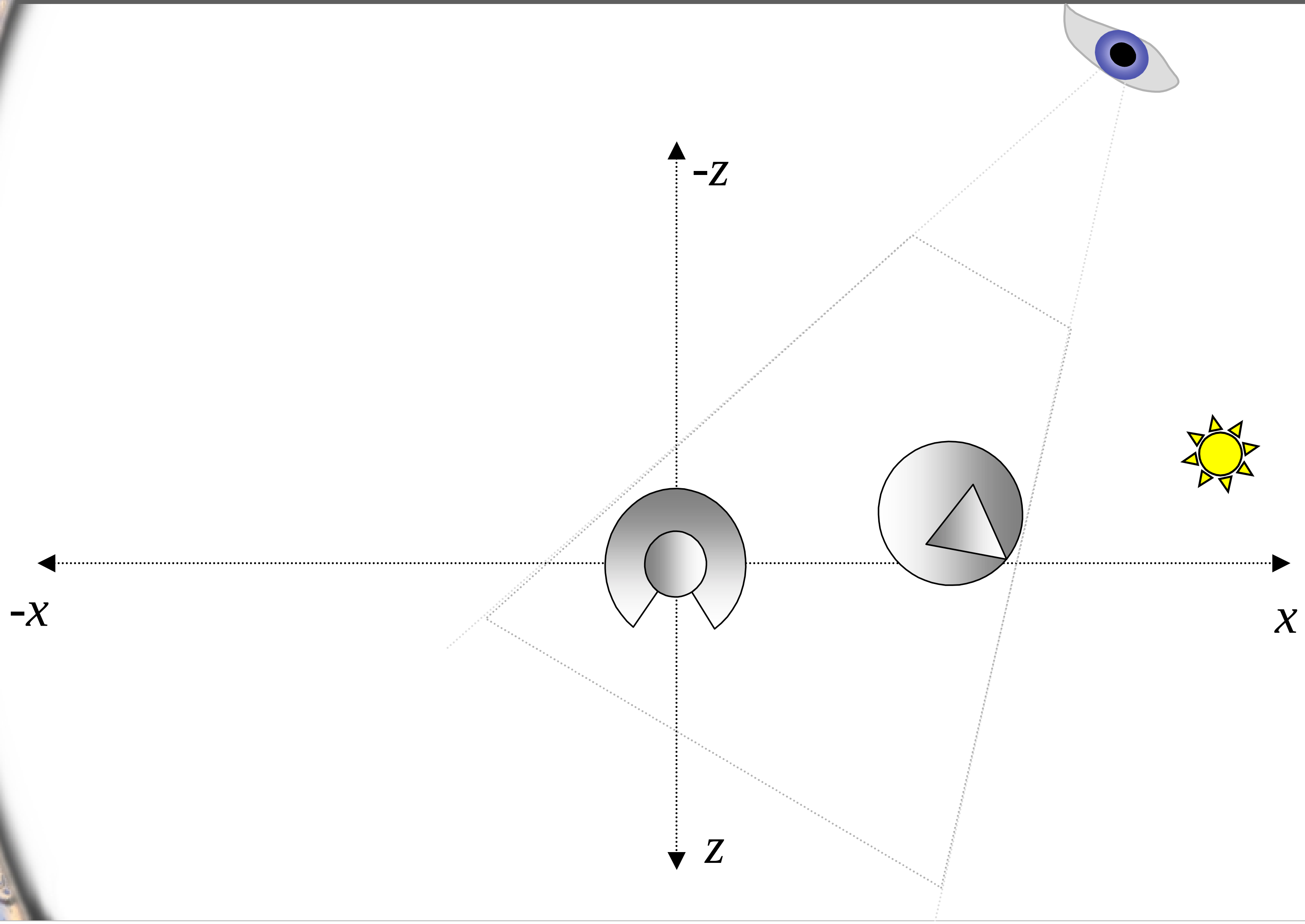
$$\boldsymbol{l}_{obj} = \mathbf{M}^{\mathrm{T}}\boldsymbol{l}_{eye}$$ infinite light source

  - **No need to transform each normal now (cheaper)**

Example Scene -- *object space* for



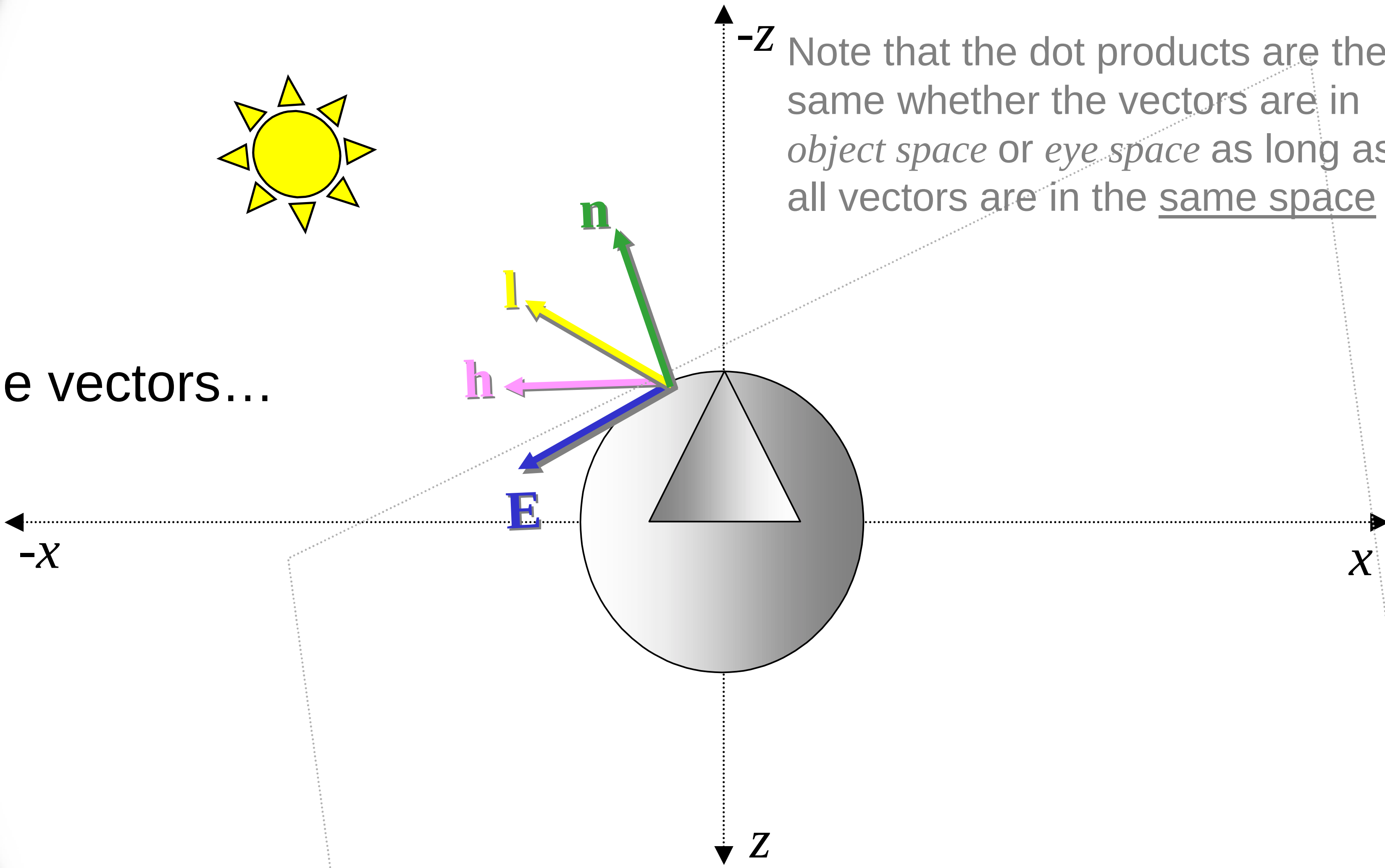
Example Scene -- *object space* for 



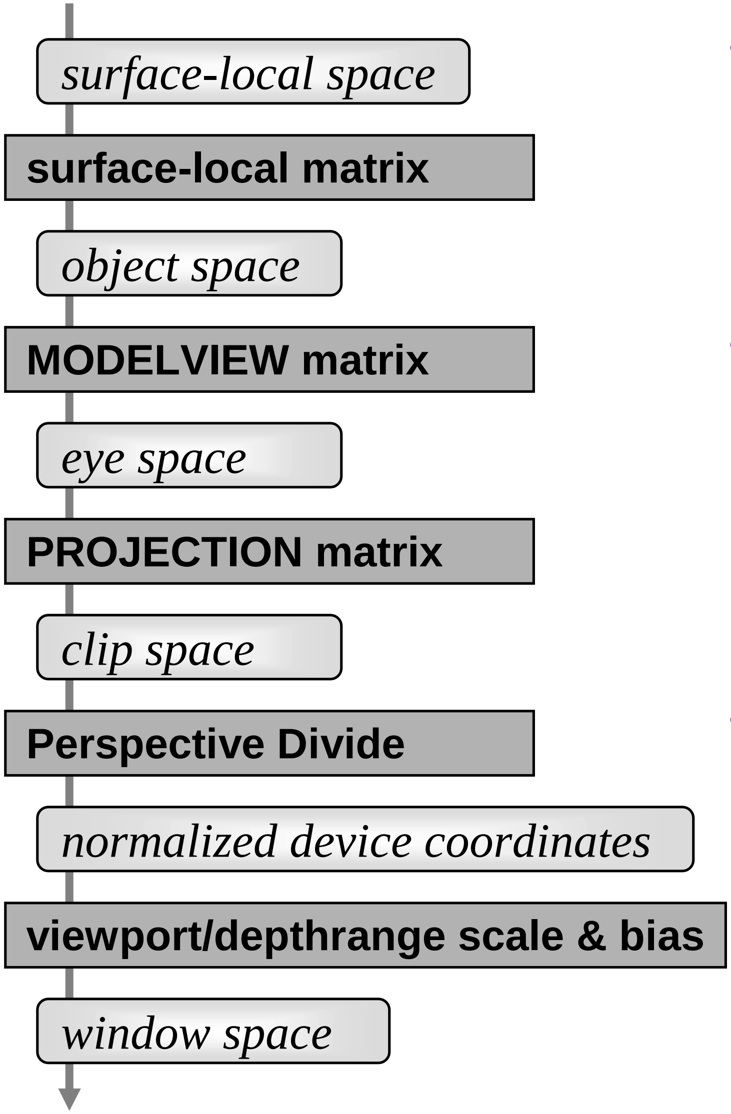
Lighting in *object space*

Note that the dot products are the same whether the vectors are in *object space* or *eye space* as long as all vectors are in the same space

The vectors…



Surface-local Space



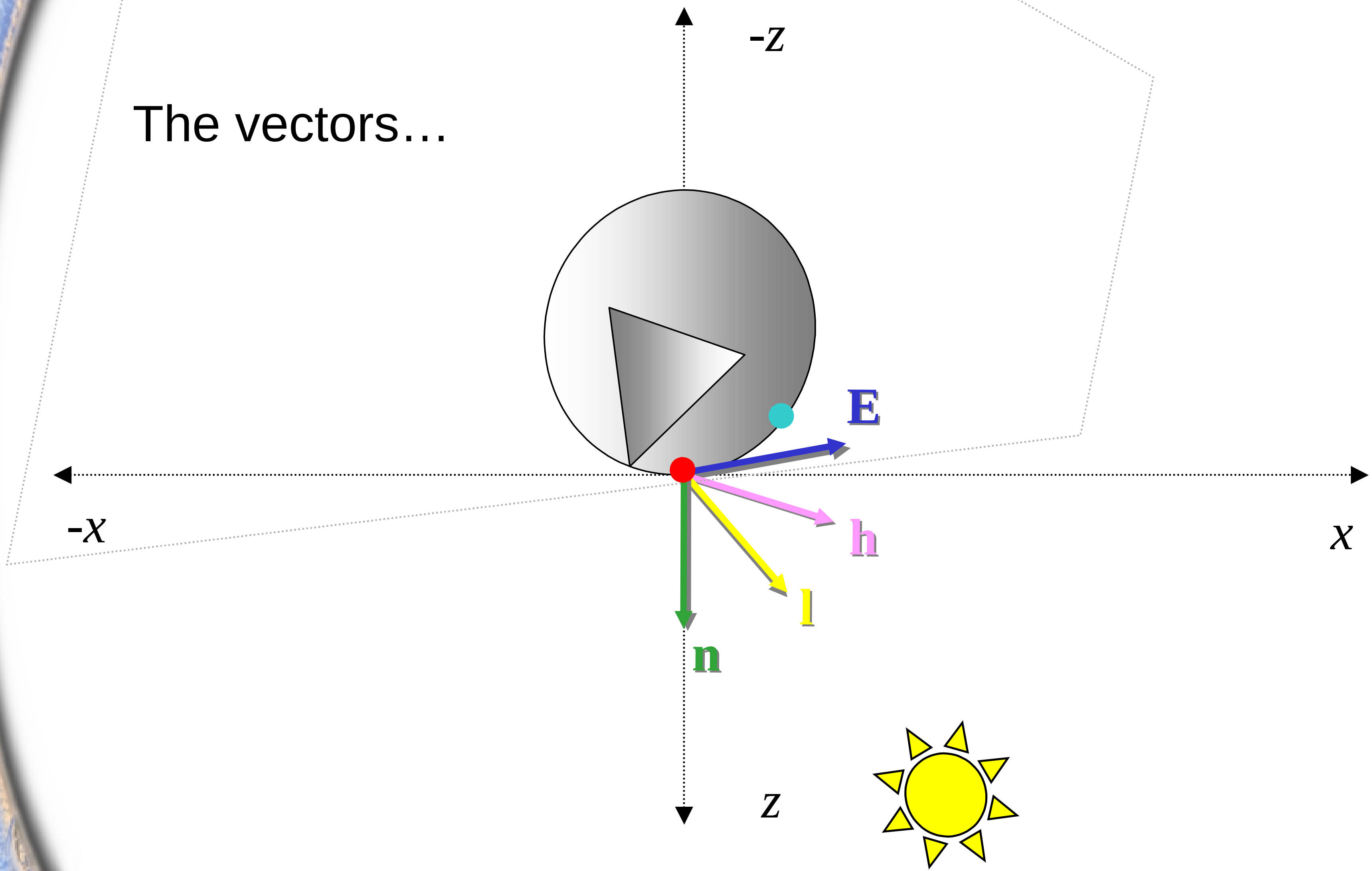
- **This gets called a lot of things…**
  - *surface-local space*
    - *tangent space*
    - *texture space*
- **A** *surface-local space* **is a class of spaces defined for every point on a surface**
- *Tangent space* **and** *texture space* **are** *surface-local spaces* **that give specific definitions to the basis vectors**
- **Consider one additional transform from** *surface-local space* **to** *object space*

Surface-local Space (2)

- **The classes of** *surface-local space* **we use are defined for every point on a surface such that the point is at the origin, and the geometric surface normal is along the positive z axis**
  - **Note that for per-pixel lighting the geometric surface normal is generally *not* what we use in the lighting equation**
- **The x and y axes are orthogonal and in the tangent plane of the surface**
- **Now the entire scene can be defined relative to any point on any surface in the scene – not just relative to any object**

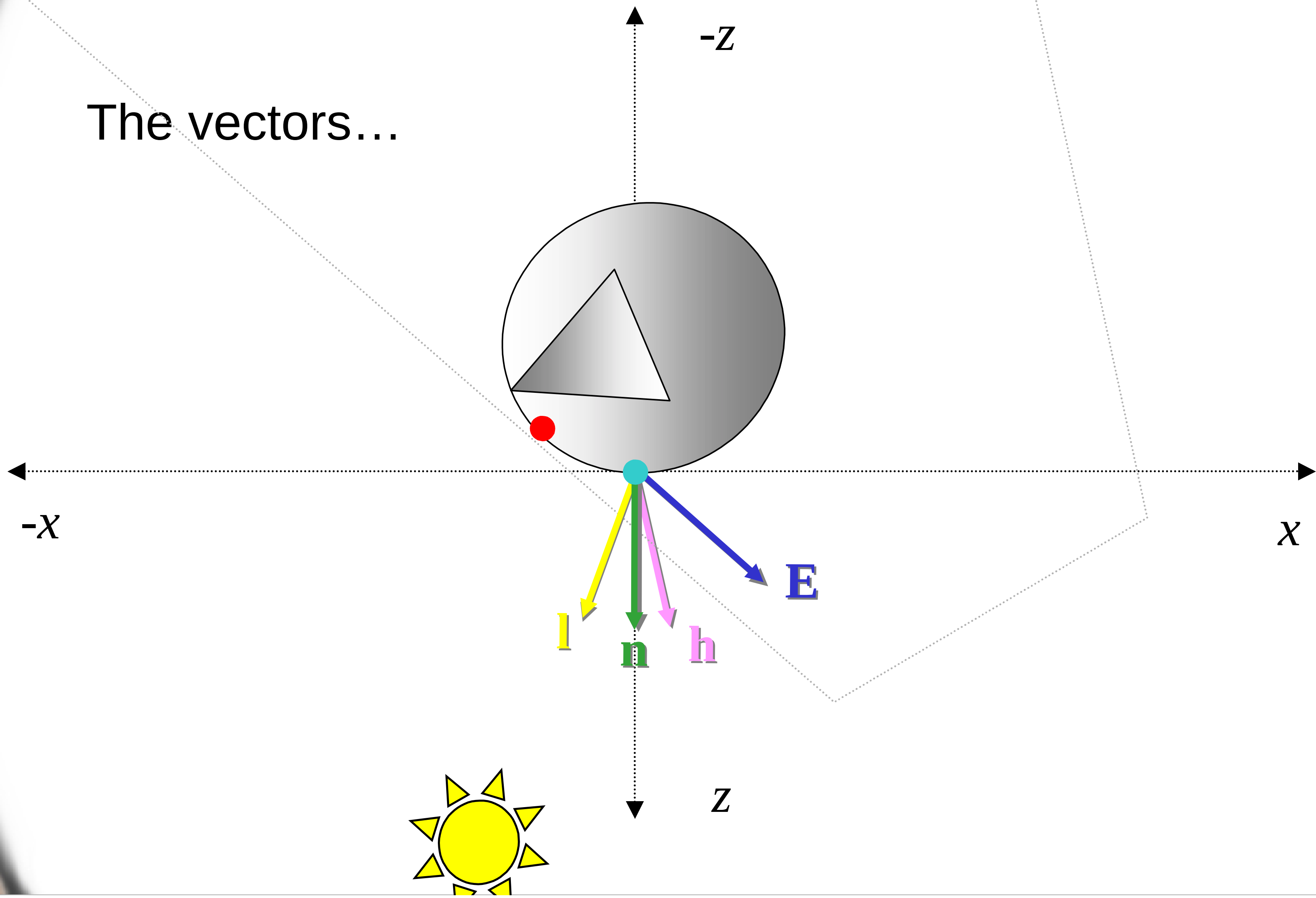
Lighting in *surface-local space*

The vectors…



Lighting in ***surface-local space***

The vectors…



Surface-local matrix

- **If we specified vertices in** *surface-local space*, **they'd all be the same!**
  - **glNormal3f(0,0,1); glVertex3f(0,0,0);**
- **The surface-local matrix,** S_l, **would provide the** *object space* **position and the** *object space* **normal orientation, and it would vary per-vertex:**

$$S_l = \begin{bmatrix} T_x & B_x & N_x & P_x \\ T_y & B_y & N_y & P_y \\ T_z & B_z & N_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T -- tangent vector
B -- binormal vector
N -- object space vertex normal
P -- object space vertex position

- **More on the tangent and binormal (*T* and *B*) vectors later…**

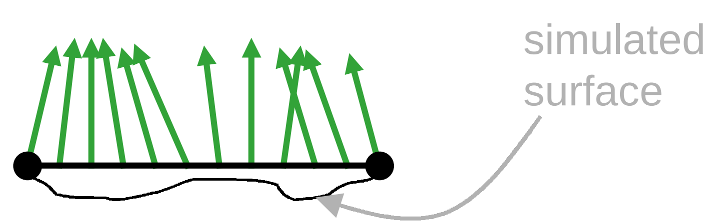
Per-Vertex Lighting in *surface-local space*

- **As with lighting in** *eye space* **or** *object space*, *surface-local space* **is a perfectly valid coordinate frame to evaluate the lighting equation**
- **We simply transform the light and eye into** *surface-local space* **– the normal is known by definition, so it doesn't need to be transformed**
- **Compare** *eye space* **and** *surface-local space* **lighting:**
  - *Eye space* **lighting: the light vector or eye vector are "free", but you must transform each normal into** *eye space*
  - *Surface-local space* **lighting: the normal is free, but you must transform the light and eye vectors into** *surface-local space*

Per-Pixel Lighting

- Getting back to the original point…
- We really want to evaluate the lighting equation per-pixel
- Rather than passing in normals per-vertex, we'll fetch them from a texture map
  - We simulate surface features with illumination only

per-vertex normals



per-pixel normals

Per-Pixel Lighting (2)

- **The texture map containing normals (normal map) clearly uses normals that are not aligned with the +z axis in** *surface-local space*
  - **This makes the tangent and binormal vectors important (see discussion later)**
- **With GeForce2 we have enough horsepower to evaluate the illumination equation at each pixel – but we don't have so much horsepower that we can do it in eye space!**
  - **That would require transforming each normal into eye space (after fetching it from the texture map)**

Per-Pixel Lighting (2)

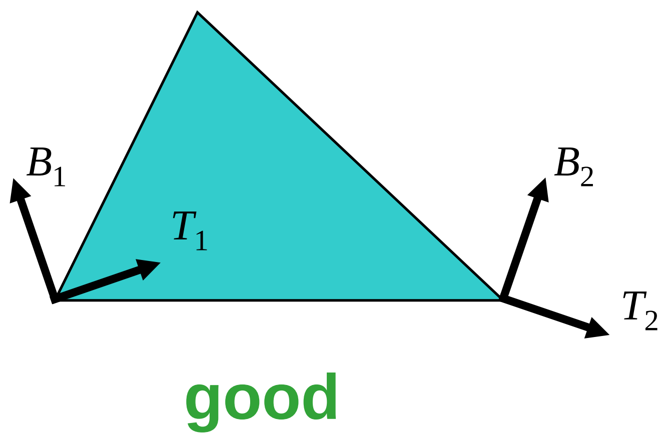
- **The better solution is to light in** *surface-local space*
  - **Fetched normals are already in the correct space**
  - **Light and eye vector interpolate nicely as long as the tangent and binormal are "well behaved"**
- **All remaining arithmetic can be evaluated with register combiners**
  - **limited range and precision not a big penalty**
- **Minor limitation: as with** *object space* **per-vertex lighting, you can't have a non-uniform scale without requiring a per-normal transform and renormalize**
  - **don't do lots of non-uniform scaling -- it won't behave correctly**

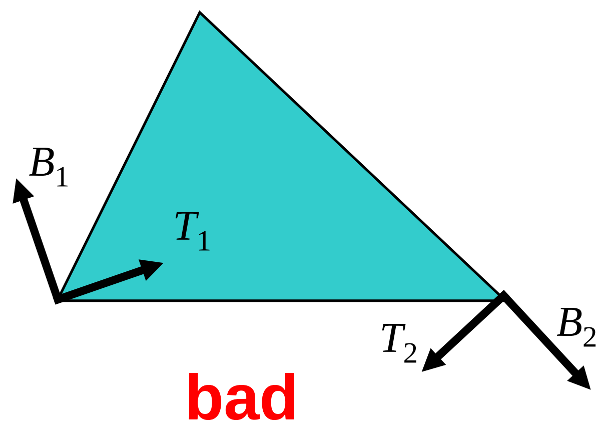
Per-Pixel Lighting (3)

- **GeForce3 *is* capable of eye space lighting per-pixel**
  - **The NV_texture_shader extension provides a 3x3 "texel matrix" that can be used to transform fetched normals from surface-local space into eye space for lighting calculations**
  - **Supports non-uniform scale with renormalization per-pixel!**
- **In addition, GeForce3 can do specular, diffuse, and decal register combiners-style per-pixel lighting in a single pass – an operation that requires 3 passes on GeForce2!**

Tangent and Binormal

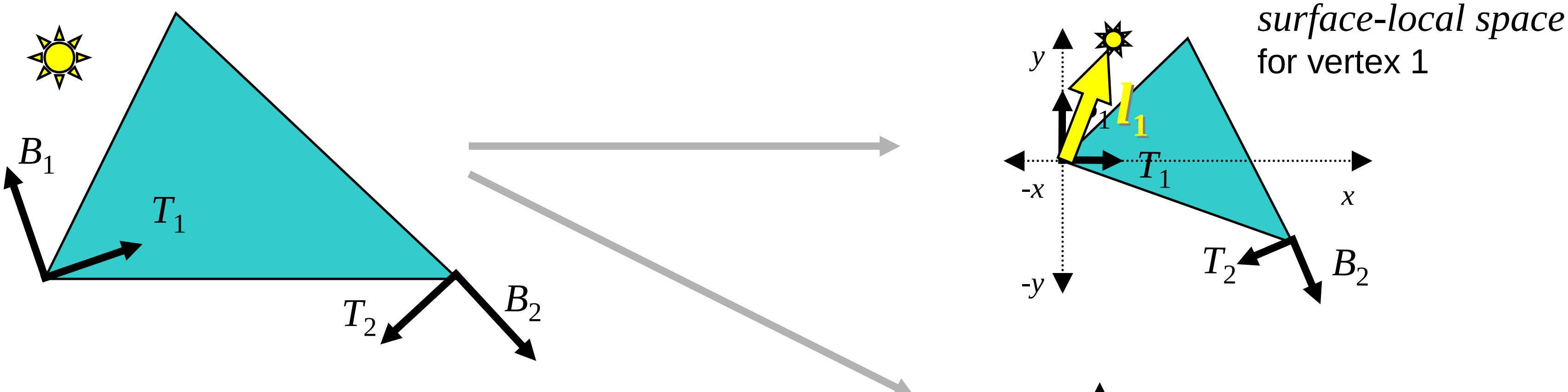
- **Whether we implement per-pixel lighting in** *surface-local space* **or** *eye space***, the tangent and binormal vectors need to be well-behaved from vertex to vertex**
- **Specifically,** $\left\| lerp(a, T_1, T_2) \right\| \approx 1$
  and $\left\| lerp(a, B_1, B_2) \right\| \approx 1$



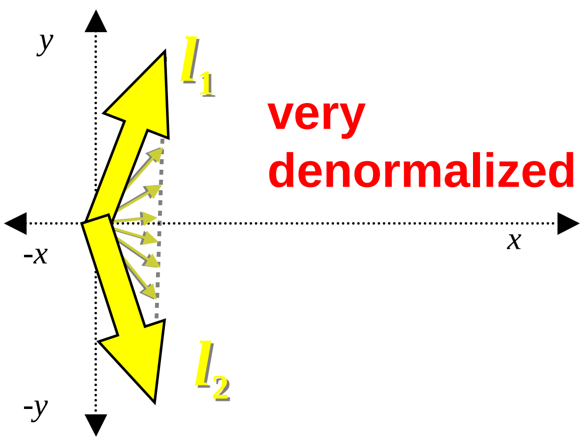


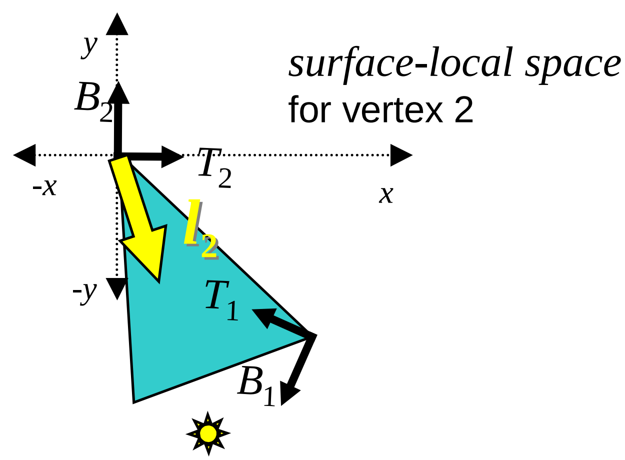
Tangent and Binormal (2)

- **Another way to look at the problem case:**



The vectors we interpolate over the polygon are:





Tangent and Binormal (3)

- In the previous case, we considered transforming the light into the *surface-local space* of each vertex and interpolating it for the per-pixel light vector -- this is what we would do for GeForce2
- For GeForce3, we can interpolate the 3x3 matrix over the surface and transform the normals by it – for this case if the tangent and binormal are not well-behaved, other anomalous behavior will result
  - Normal “twisting”
  - Incorrect bump scale/smoothing
  - The interpolated matrix should be “nearly orthonormal”

Implementation topics

- **Please check out the NVIDIA OpenGL SDK presentations for information on these related topics:**
  - **How do you compute a** *texture space* **surface-local matrix for textured polygonal models?**
  - **How do you animate per-pixel shaded surfaces?**

Questions?

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www.nvidia.com/Developer